13.2 Calculus on 3D Curves

2D Example: Consider

$$x = t, y = 2 - t^2$$

which can also be written as

$$\boldsymbol{r}(t) = \langle t, 2 - t^2 \rangle$$

Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

When $t = 1$…

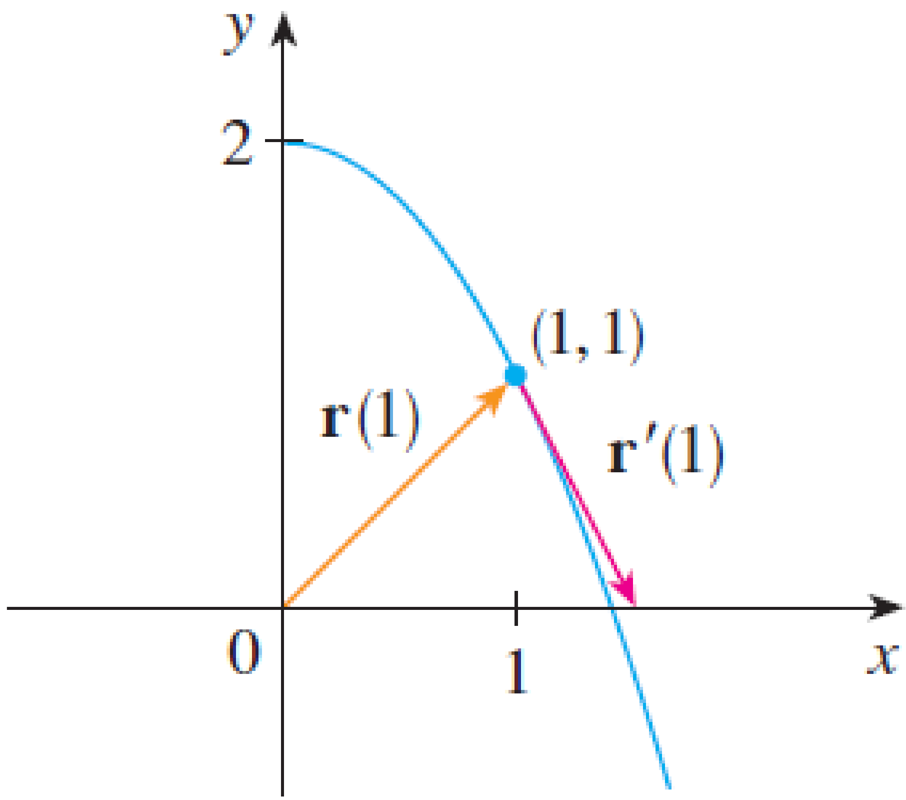
Find the location.

Find the slope of the tangent line.

Find a vector in the direction of the tangent line.

Visual of last example:

$$\boldsymbol{r}(t) = \langle t, 2 - t^2 \rangle$$



***In general:* Vector Calculus**

For $\vec{\boldsymbol{r}}(t) = \langle x(t), y(t), z(t) \rangle$, we define

$$\vec{\boldsymbol{r}}'(t) = \lim_{h \to 0} \left\langle \frac{x(t+h) - x(t)}{h}, \frac{y(t+h) - y(t)}{h}, \frac{z(t+h) - z(t)}{h} \right\rangle$$

which is the same as

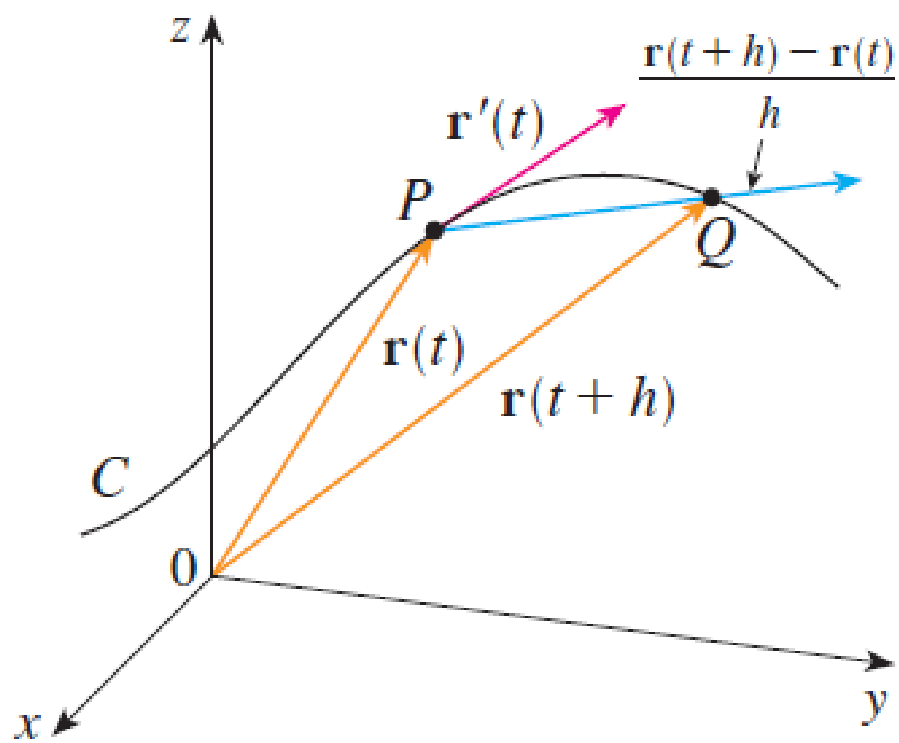
$$\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

And

$$\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

is a tangent vector to the curve.

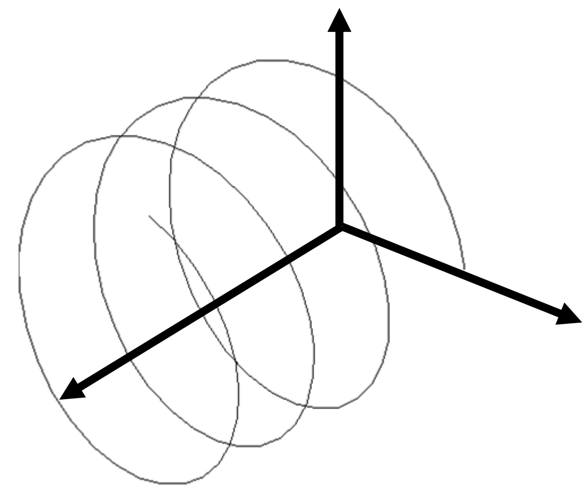
Do calculus **component-wise**!



Example

$\vec{\boldsymbol{r}}(t) = \langle t, \cos(2t), \sin(2t)\rangle$.

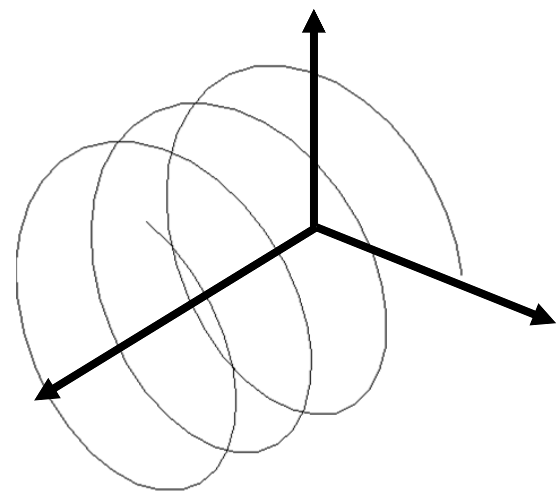
1. Find $\vec{\boldsymbol{r}}'(t)$.
2. Find $\vec{\boldsymbol{r}}(0)$ and $\vec{\boldsymbol{r}}(\pi/4)$.
3. Find $\vec{\boldsymbol{r}}'(0)$ and $\vec{\boldsymbol{r}}'(\pi/4)$.
4. Find the unit tangent vector $\vec{\boldsymbol{T}}(t)$ at $t = \pi/4$.



Example Continued

$\vec{\boldsymbol{r}}(t) = \langle t, \cos(2t), \sin(2t) \rangle.$

5. Find parametric equations for the tangent line at $t = 0$.
6. Find parametric equation for the tangent line at $t = \pi/4$.



Example: Antiderivatives

First some review

Find the antiderivative of

$$f'(t) = \sin(t) + e^{2t} - \frac{t^3}{5}$$

with $f(0) = 7$.

Now find the antiderivative of

$$\vec{\boldsymbol{r}}'(t) = \langle e^{3t}, t^4, \sin(t) - t \rangle.$$

with $\vec{\boldsymbol{r}}(0) = \boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}.$

3D calculus from today

$\vec{\boldsymbol{r}}'(t) = \langle x'(t), y'(t), z'(t)\rangle$ *tangent vector* (13.2)

$\vec{\boldsymbol{T}}(t) = \frac{1}{|\vec{\boldsymbol{r}}'(t)|}\vec{\boldsymbol{r}}'(t)$ *unit tangent vector* (13.2)

$\int \vec{\boldsymbol{r}}(t)dt = \left\langle \int x(t)dt, \int y(t)dt, \int z(t)dt \right\rangle$ $antiderivative\ vector\ \left(\frac{13.2}{4}\right)$

To find the tangent line to $\vec{\boldsymbol{r}}(t)$ at $t = t_0$

Step 1: Compute $\vec{\boldsymbol{r}}(t_0) = \langle x(t_0), y(t_0), z(t_0)\rangle$. Use as $\langle x_0, y_0, z_0\rangle$.

Step 2: Compute $\vec{\boldsymbol{r}}'(t_0) = \langle x'(t_0), y'(t_0), z'(t_0)\rangle$. Use as $\langle a, b, c\rangle$.

Step 3: $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$